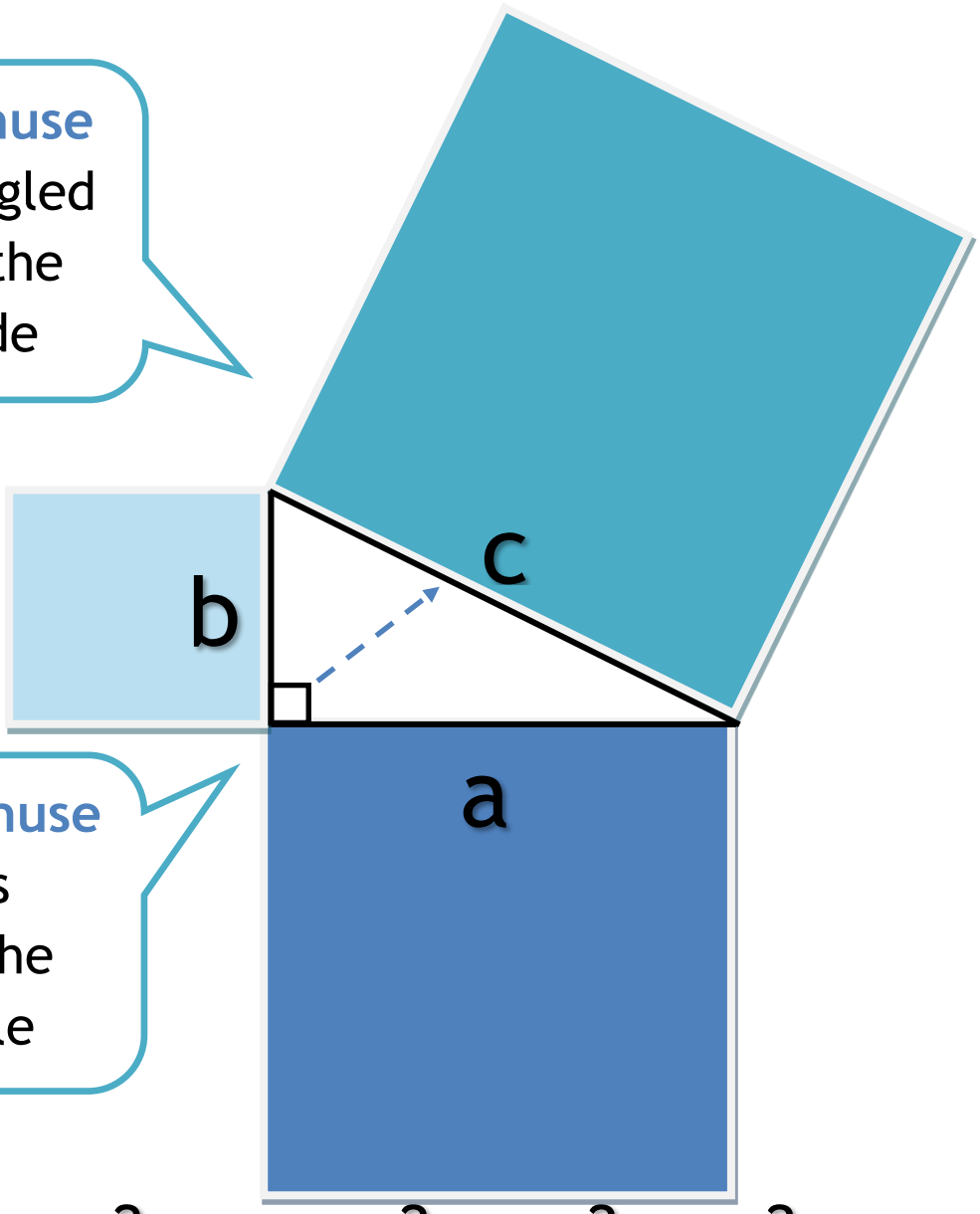
Pythagoras' Theorem

The **hypotenuse** of a right-angled triangle is the longest side



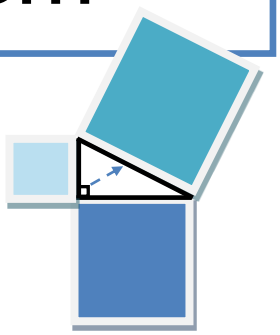
The **hypotenuse** is always opposite the right-angle

$c^2 = a^2 + b^2$ or $a^2 = c^2 - b^2$

or $b^2 = c^2 - a^2$

The area of the square on the hypotenuse is equal to the sum of the squares on the other two sides

Pythagoras' Theorem



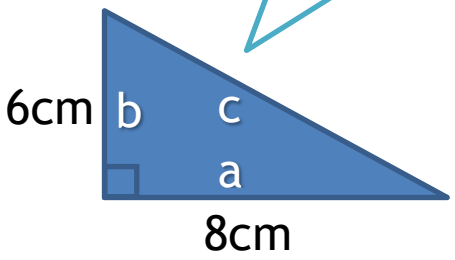
Find the length of the missing sides

$c^2 = a^2 + b^2$

$b^2 = c^2 - a^2$

$a^2 = c^2 - b^2$

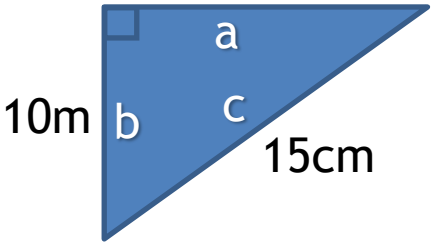
You add to find the hypotenuse



$c^2 = a^2 + b^2$
$c^2 = 8^2 + 6^2$
$c^2 = 64 + 36$
$c^2 = 100$
$c = \sqrt{100}$
$c = 10\text{cm}$

1. Label the triangle
2. Decide whether to + or -
   + for hypotenuse
   - for a shorter side
3. Square the numbers
4. Square root your answer
5. Put on the correct units

You subtract to find a shorter side

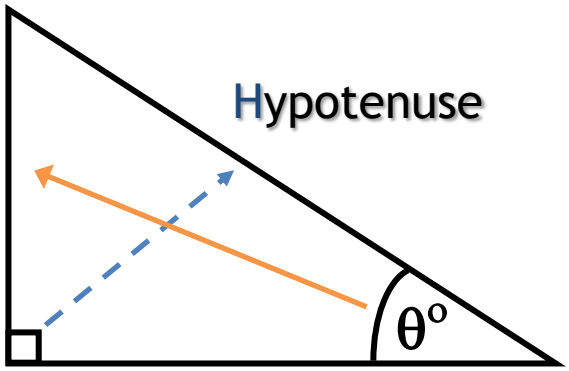


$a^2 = c^2 - b^2$
$a^2 = 15^2 - 10^2$
$a^2 = 225 - 100$
$a^2 = 125$
$a = \sqrt{125}$
$a = 11.2\text{m}$ (1dp)

If you don't have a calculator, leave your answer as a square root

Trigonometry

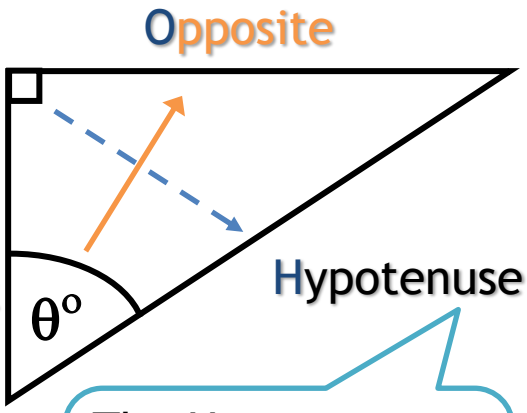
The study of triangles is called Trigonometry



The Hypotenuse is the longest side

The side opposite the angle θ

θ is the angle you are going to use



The side next to the angle θ

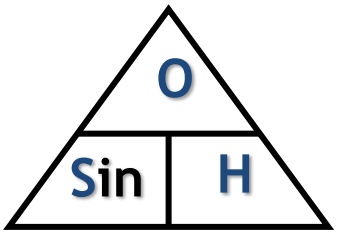
The Hypotenuse is always opposite the right-angle

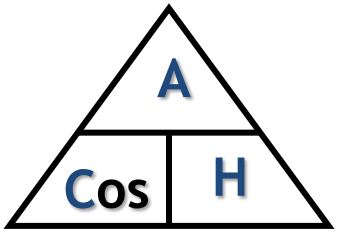
$$S\frac{O}{H}H \qquad S = \frac{O}{H}$$

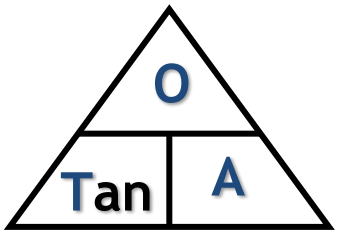
$$C\frac{A}{H}H \qquad C = \frac{A}{H}$$

$$T\frac{O}{A}A \qquad T = \frac{O}{A}$$

$$\text{Sin } \theta = \frac{\text{Opposite}}{\text{Hypotenuse}} \qquad \text{Cos } \theta = \frac{\text{Adjacent } \theta}{\text{Hypotenuse}} \qquad \text{Tan } \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$





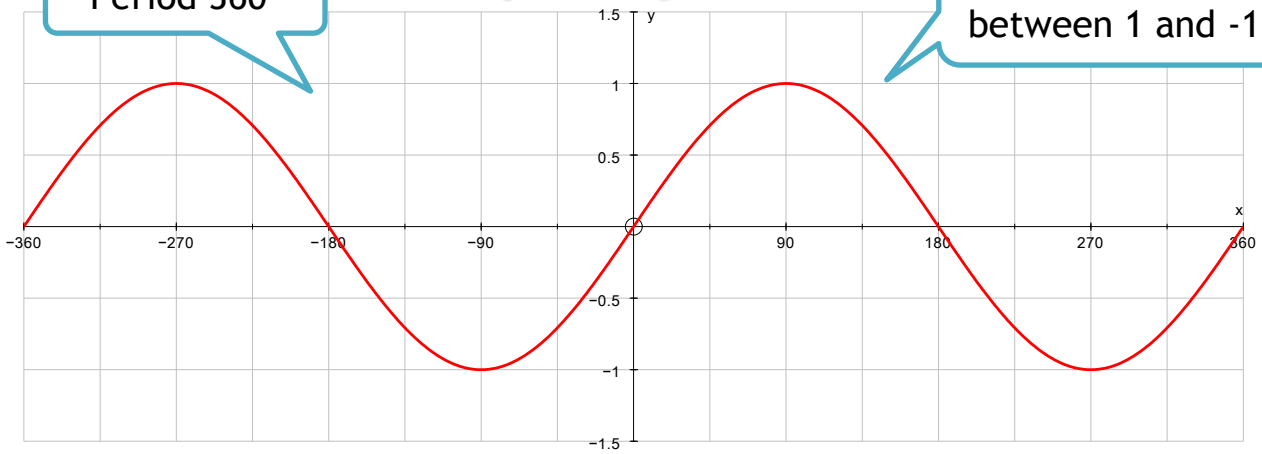


Trigonometry

Graph of $y = \sin x$

Period 360°

Oscillates between 1 and -1



Graph of $y = \cos x$

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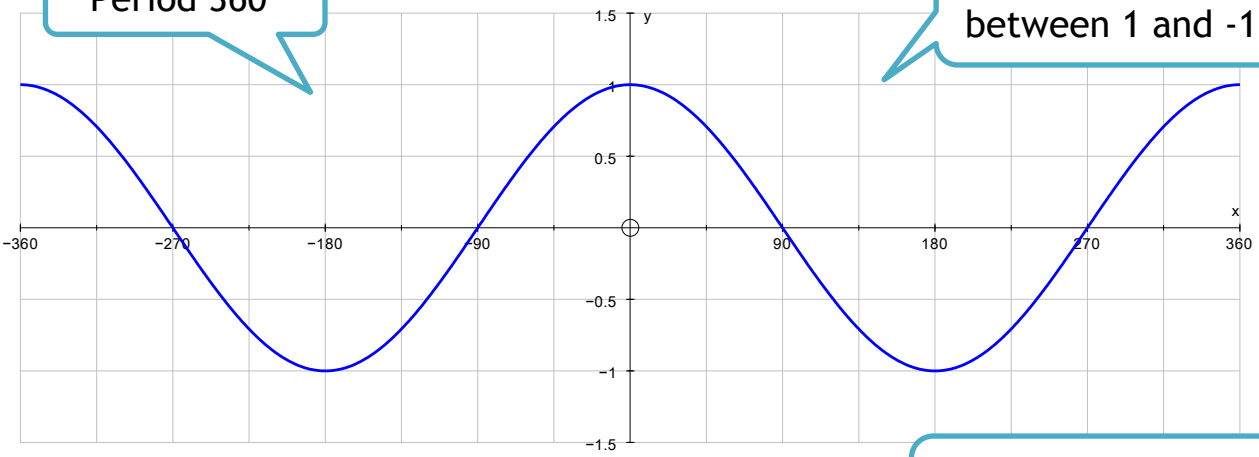
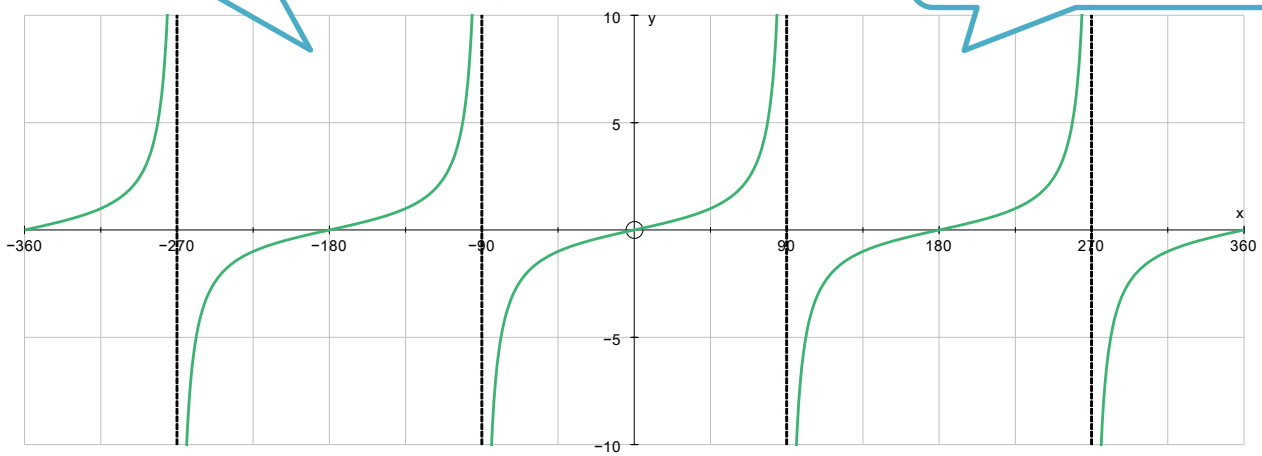


Graph of $y = \tan x$

Period 180°

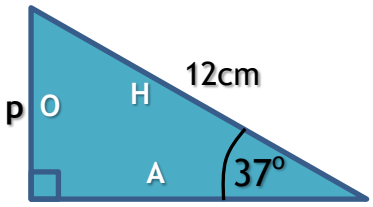
Asymptotes at -270°, -90°, 90°, 270°



Trigonometry

Example 1

Find the length of side p.



$S^OH\ C^AH\ T^OA$

$\text{Sin} = \frac{O}{H}$

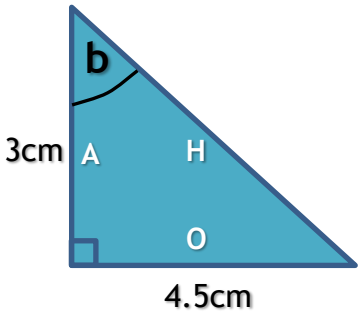
$\text{Sin } 37^\circ = \frac{p}{12}$

$12 \times \text{Sin } 37^\circ = p$

$p = 7.2\text{cm}$ (1dp)

Example 2

Find the size of angle b.



$S^OH\ C^AH\ T^OA$

$\text{Tan} = \frac{O}{A}$

$\text{Tan } b = \frac{4.5}{3}$

$b = \text{Tan}^{-1}\frac{4.5}{3}$

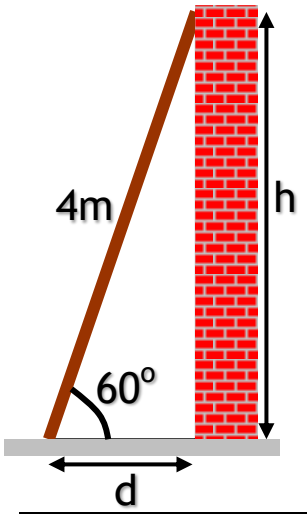
$b = 56.3^\circ$ (1dp)

Method

1. Label the sides, O, A, H.
2. Write down $S^OH\ C^AH\ T^OA$
3. Decide which two sides you are using, then select Sin, Cos or Tan
4. Write the trig. function with the information for your triangle.
5. Calculate.
6. Check your answer is sensible.

Example 3 A ladder of 4 metres in length is resting against a wall. The ladder makes an angle of 60° with the ground.

(a) Find how high up the wall the ladder reaches, h.
(b) Find the distance from the wall to the bottom of the ladder, d.



(a) $S^OH\ C^AH\ T^OA$

$\text{Sin} = \frac{O}{H}$

$\text{Sin } 60^\circ = \frac{h}{4}$

$4 \times \text{Sin } 60^\circ = h$

$h = 3.46\text{m}$ (2dp)

(b) $S^OH\ C^AH\ T^OA$

$\text{Cos} = \frac{A}{H}$

$\text{Cos } 60^\circ = \frac{d}{4}$

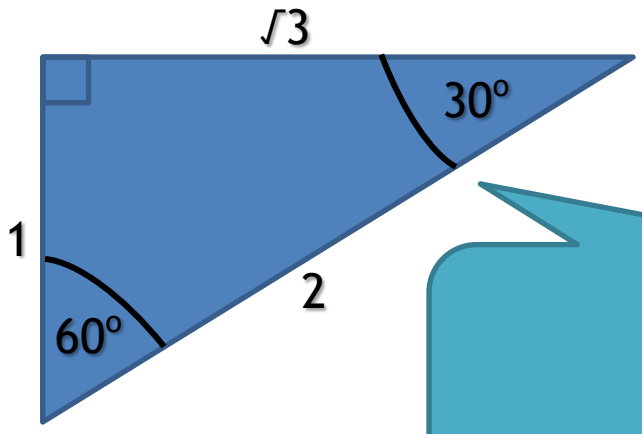
$4 \times \text{Cos } 60^\circ = d$

$d = 2\text{m}$

Once you know h you could use Pythagoras rule to calculate d

Trigonometry

$S^{O}H\ C^{A}H\ T^{O}A$



Learn: Sin 30° = $\frac{1}{2}$

Sin 30° = $\frac{1}{2}$ = $\frac{OPP}{HYP}$

OPP = 1 HYP = 2

Using Pythagoras ADJ must = √3

Sin 30° = $\frac{1}{2}$

Cos 60° = $\frac{1}{2}$

Sin 60° = $\frac{\sqrt{3}}{2}$

Cos 30° = $\frac{\sqrt{3}}{2}$

Tan 60° = $\frac{\sqrt{3}}{1}$ = $\sqrt{3}$

Tan 30° = $\frac{1}{\sqrt{3}}$

Learn: Tan 45° = 1 = $\frac{1}{1}$

You must label the triangle O, A, H for the angle you are using.

Changing the angle will mean repositioning the labels

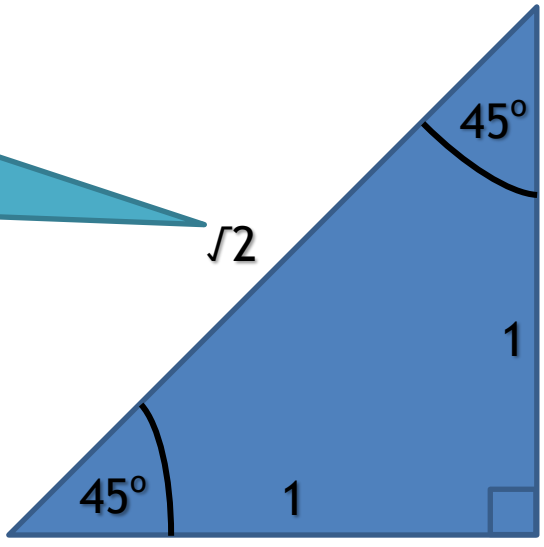
Tan 45° = 1 = $\frac{1}{1}$ = $\frac{OPP}{ADJ}$

OPP = 1 ADJ = 1

Using Pythagoras HYP must = √2

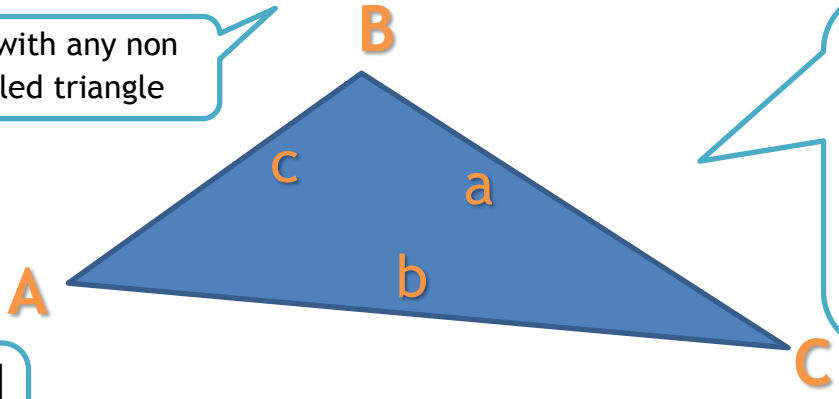
Sin 45° = $\frac{1}{\sqrt{2}}$

Cos 45° = $\frac{1}{\sqrt{2}}$



The Cosine Rule

It is used with any non right-angled triangle



ANGLES are marked with CAPITALS and the opposite side with the same lower case letter

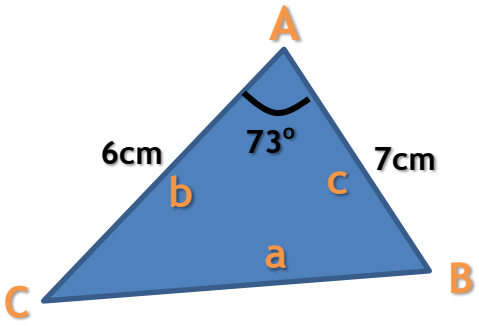
To find a side

$$a^2 = b^2 + c^2 - 2bc\cos A$$

or $$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

To find an angle

Use this rule when given two sides and the angle between them or all three sides and no angles



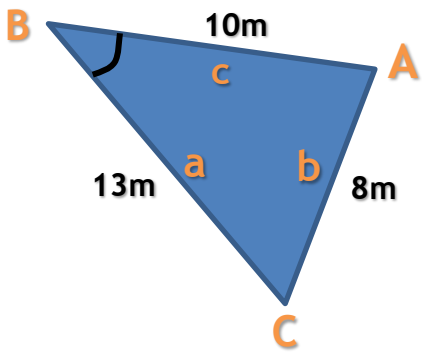
Example 1 - Find the length of side BC

Use $a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = 6^2 + 7^2 - 2 \times 6 \times 7 \times \cos 73°$

$a^2 = 60.4407768$

$a = \sqrt{60.4407768} = 7.8\text{cm}$ (1dp)



Example 2 - Find angle B

Use $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\cos B = \frac{13^2 + 10^2 - 8^2}{2 \times 13 \times 10}$

$\cos B = \frac{205}{260} = 0.788461538$

INV cos

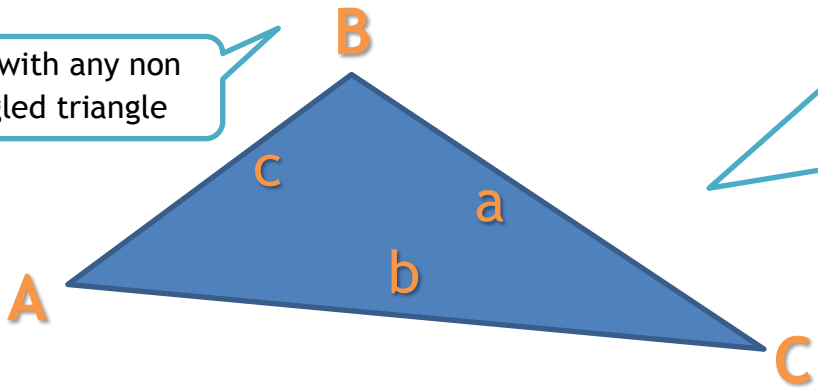
$B = \cos^{-1}(0.788461538)$

$B = 38.0°$ (1dp)

Use full calculator values to the end

The Sine Rule

It is used with any non right-angled triangle



ANGLES are marked with CAPITALS and the opposite side with the same lower case letter

To find a side put the sides on top

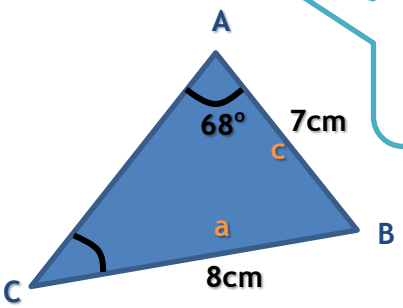
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

or

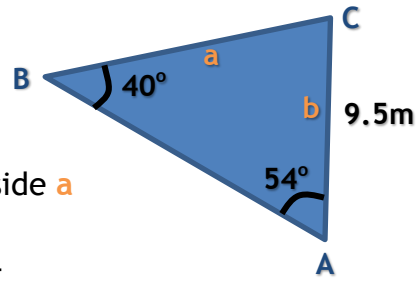
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

To find an angle put the angles on top

Use this rule when you can partner one angle and side and need to find the angle or side from another pair



Partners are opposite each other



Example 1 - Find angle C

Use $\frac{\sin C}{c} = \frac{\sin A}{a}$

$\frac{\sin C}{7} = \frac{\sin 68^\circ}{8}$

$\sin C = \frac{7 \times \sin 68^\circ}{8}$

$\sin C = 0.811285872$

$C = \sin^{-1}(0.811285872)$

$C = 54.2^\circ$ (1dp)

INV sin

Use full calculator values to the end

Example 2 - Find side a

Use $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{a}{\sin 54^\circ} = \frac{9.5}{\sin 40^\circ}$

$\frac{9.5 \times \sin 54}{\sin 40^\circ}$

$a = 12.0\text{m}$ (1dp)