The inside story

Polygons

BOSS — Gareth, crop polygons have been appearing in the farmlands. I want the inside story on this and I want it now! Or, at least, soon. I'm a reasonable boss and I want this article to be well researched and not just sensationalist clickbait.

GARETH — Sure thing, boss!

And here I am walking around the exterior of one of the pentagons. Yeah, 5 sides. I couldn't get the angle at first but I realised that when I went around once, I was back where I started, facing the same direction. All of these exterior angles must add to 360°.

Here's me on an octagon, dodecagon…

BOSS — OK, OK, nice work. Did you get the inside story?

GARETH — Oh yeah, I got inside, to the interior you know, it was really quite mystical.

But here's the thing: I'd always get back to the start, but for some polygons I found myself facing the same direction and for others the opposite direction. The sums were different for different polygons.

BOSS — Illuminati?

GARETH — Luckily no but get this: choose a polygon and a vertex on it. Cut the polygon into triangles from that vertex to other vertices. The interior angles of these triangles make up the interior angles of the polygon. We all know that the interior angles of a triangle add up to 180°.

BOSS — Yes, of course.

GARETH — This hexagon gets cut into 4 triangles, so the sum of the interior angles must be $4 \times 180 = 720°$.

BOSS — And the others?

GARETH — Well, this goes all the way up.

GARETH	Look! The number of triangles is two less than the number of sides.
BOSS	And so the sum of the angles is 180 multiplied by that. Great Scott!
GARETH	And do you know what really blows this thing wide open? For a regular polygon…
BOSS	Hey! This isn't a regular column, just a one-off assignment kid.
GARETH	No, regular polygons have all interior angles equal to each other and all exterior angles equal to each other too. So if I want an interior angle of a regular polygon, I just divide the angle sum by the number of sides, and the same for exterior angles! Look, let's do it for a nonagon. The sum of the interior angles is $180 \times 9 - 2 = 1260°$. The interior angles are $1260° \div 9 = 140°$. The exterior angles are $360° \div 9 = 40°$.
BOSS	Great work for an intern, kid. Maybe one day I'll pay you.
GARETH	No problem boss. It was a walk in the park.